MODELLING THE DETERMINANTS OF NAIRA/US DOLLAR CURRENCY EXCHANGE RATES USING PRINCIPAL COMPONENT ANALYSIS (PCA) AND SINGULAR VALUE DECOMPOSITION (SVD)

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ABSTRACT

In the literature, a large number of factors are mentioned as being responsible for the relative strength or weakness of a country's currency with respect to the currencies of other nations. Many of these variables are usually highly correlated with each other leading to the twin problems of multicollinearity and redundancy in the explanatory variables.This study examines the main determinants of currency exchange rates, and applies Principal Components Analysis (PCA) and Singular Value Decomposition (SVD) to Naira/USD data regarding the factors responsible for the steady deterioration in the Naira/USD exchange rates over time. The aim is to rank the factors in order of importance and impact on the Naira/USD exchange rate in the Nigerian environment, as such ranking may not be of universal application in all countries. The study uses Machine Learning algorithms to achieve dimensionality reduction and thus address the problem of multicollinearity and high dimensionality. The study found that three principal components adequately explain more than seventy percent of the variance, thereby making it unnecessary to use more than three explanatory variables in similar studies predicting the evolution of currency exchange rates. In this study the number of explanatory variables was drastically reduced from twenty-one to just three, while solving the problem of multicollinearity.

Keywords: Exchange rates, Dimensionality reduction, Machine Learning, Principal components.

1. INTRODUCTION

Currency exchange rate fundamentally influences the direction of economic activities in a country. It is an important variable that affects the performance of other macro-economic variables in achieving economic growth and development (Oke, 2018).

In the literature, many factors are mentioned as responsible for the relative strength or weakness of a nation's currency with respect to the currencies of other nations. Some of these factors include, but are not restricted to, the GDP, Inflation, Interest rates, Import cover, Exports, External reserves, Balance of Payments, Deposit and Lending rates, etc., among so many others (Rodrigues, 2020 ; Oke et. al. 2018).

Many of these variables tend to be highly correlated with each other resulting in the problem of multicollinearity. A related problem is the issue of redundancy in the many related variables amid what has been described as the "curse of dimensionality"(Rahayu et al, 2017). This is the case where the number of variables is so large that, for meaningful analysis to be carried out, there is usually a need to reduce the number of variables in the analysis and focus only on the most important ones without losing much important information.This study aims to identify the optimum number of explanatory variables to use when modelling Naira/USD currency exchange rates. The objective here is to reduce the very large number of usually highly correlated independent variables mentioned in the currency exchange rate literature to a much smaller and manageable number of explanatory variables for meaningful analysis without losing much of the important information in the data set, while at the same time addressing the issue of multicollinearity.

According to Rahayu et. al (2017), the term Multicollinearity was first discovered by Frisch (1934) and indicates perfect linear relationship among some or all of the independent variables of a multiple regression model. Gujarati (2003) averred that multicollinearity can arise from variables influenced by the same factors. It can also arise from the use of lagged models e.g. strong correlation between y_t and y_{t-1}, from data collection methods, as well as from overdetermined models I.e. where the number of explanatory variables is greater than data observations.

Detecting multicollinearity can be achieved by calculating correlation coefficients among independent variables (e.g.where simple correlation coefficient exceeds 0.8). Multicollinearity can also be detected by calculating Variance Inflation Factor (VIF). For example where VIF > 10

Tolerance (TOL) is also used calculate the size of multicollinearity e.g. where Tolerance < 0.1

$$TOL_i = \frac{1}{VIF_i} = 1 - {R_i}^2$$

Other measures for calculating multicollinearity include Eigenvalues and Conditions Index (CI). The impact of Multicollinearity can be addressed through Ridge Regression or PCA (both could be done to confirm ranking)

2. REVIEW OF LITERATURE

Currency exchange rate is a major indicator of a country's level of economic health (Rodrigues, 2020). Other important economic indicators include inflation, interest rates, consumer price index and money supply. Volatile exchange rates could make it difficult to ascertain or forecast the value of goods and services in an economy thus resulting in an unstable economic environment that might militate against stable economic growth (Nwude, 2012; Ajao and Igbokoyi, 2013). Many macroeconomic variables tend to be strongly correlated with each other, thereby manifesting the problem of multicollinearity (Rahayu et. al.,2017). This usually results in redundancy of variables leading to situations where researchers find themselves having to contend with too many explanatory variables in their analyses.

Rahayu et al (2017) used Principal component analysis to reduce multicollinearity in exchange rates of some Asian countries. They found that three main components had sufficient predictive power in their model. Oke and Adetan (2018) carried out an empirical analysis of the determinants of exchange rate in Nigeria. They used the Bound Test to find that Gross Domestic Product (GDP), Interest rate and Inflation had positive effects on the exchange rate in Nigeria. Regos (2015) modeled exchange rates using price levels and country risk in seven countries. The author concluded that sovereign risk has significant effect on exchange rates. Agu (2002) in Oke et. al. (2018) posits that the mechanism of exchange rate determination and the different systems of managing the exchange rate of a nation's currency should be properly done to achieve efficient allocation of scarce resources for growth and development. Jhingan (2005) concluded that a country has to control its exchange rate in order to maintain internal and external balance.

3. MATERIALS AND METHODS

3.1 Data

Monthly data on Exchange Rates, Balance of Payments, Inflation, Interest Rates, External Reserves, Deposit and Lending Rates, and other macro-economic variables mentioned in the literature as contributing to the determination of exchange rates were obtained from the Central Bank of Nigeria data base from 2008 to 2021 inclusive.

3.2 Methods

Exploratory Data Analysis is carried out by Machine Learning tools using Python codes. Principal Component Analysis (PCA) is then deployed using the same Machine Learning and Python computer package. Principal Component Analysis (PCA) is a powerful statistical instrument often deployed to overcome the problems of high dimensionality and multicollinearity among the explanatory variables.

The objective of PCA is to obtain most of the important information present in the data even as the number of dimensions is reduced. This addresses the issue of what is known as the "curse of dimensionality". The curse of dimensionality is when we have more dimensions than required i.e. too many dimensions or explanatory variables. More dimensions in a model can introduce multicollinearity and overfitting.

When a model overfits, such model might fail to perform or predict in the presence of new or previously unseen data. PCA helps reduce redundancy in the dimensions.

3.2.1 How PCA works.

For two dimensional space, PCA essentially rotates the coordinate axes, such that one axis captures almost all the information content or variance previously present in both axes and the second axis can be discarded, thus reducing the number of dimensions from two to one while retaining most of the important information. Here two independent parameters X_1 and X_2 are fed into the model. In Python implementation, use:

$$\text{Model.fit}(X1, X2)$$

The model is initially capturing only the individual pieces of information available in the explanatory variables and not the joint spread which is far richer as it tells how the two variables vary together. The goal of PCA is to capture this covariance information to enrich the model. PCA is carried out in several steps:

Step 1: Standardize the independent variables.

This is achieved by applying Z-score to the data. This centers the data points to the origin.

$$Z = \frac{(X_i - \bar{x})}{\sigma}$$

The z-score of X_i indicates how many standard deviations away this X_i value is from the central value or average.

When $X_i > \bar{x}$, z-score is positive

When $X_i < \bar{X}$, z-score is negative

At origin, z-score is 0.

Step 2: Generate the covariance matrix for each dimension. This captures all the covariance information among the dimensions.

In the original two dimensional space, the data has averages $\bar{X}_1$ and $\bar{X}_2$ as well as covariance between X_1 and X_2.

On standardizing the data points, the central values become the dimensions and the data is spread around them.

Covariance between X_1 and X_2 is presented as a matrix:

$$\begin{bmatrix} cov(X,X) & cov(X,Y) & cov(X,Z) \\ cov(Y,X) & cov(Y,Y) & cov(Y,Z) \\ cov(Z,X) & cov(Z,Y) & cov(Z,Z) \end{bmatrix} \rightarrow \begin{bmatrix} {\sigma_1}^2 & 0 & 0 \\ 0 & {\sigma_2}^2 & 0 \\ 0 & 0 & {\sigma_P}^2 \end{bmatrix}$$

$$AA^T \qquad\qquad BB^T$$

This matrix shows numerically the amount of information contained in the two dimensional space between X_1 and X_2.

The diagonal elements represent the variance of X_1 with itself and X_2 with itself indicating how much information is contained within each variable.

The off-diagonal elements show the correlation between X_1 and X_2 i.e. the interaction with each other. This information serves as input for the model.

Step 3: Eigen Decomposition

Eigen decomposition transforms the original covariance matrix between X_1 and X_2 into the identity matrix.This matrix is empty, as all the information has been absorbed by the axis. The process yields two outputs- eigenvectors and eigenvalues. The eigenvectors are the new dimensions of the new mathematical space while eigenvalues indicate the information content of each eigenvector. This is the spread or the variance of the data on each eigenvector.

Step 4: Sort corresponding eigenvectors and eigenvalues.Then multiply each matrix with its transpose to obtain the covariance matrix.

The covariance matrix shows the amount of information contained in the mathematical space, and can be visually displayed on a pairplot using sns.pairplot() in Python language.The diagonal elements in the pairplot show the relationship between each variable and itself. The off-diagonal elements describe the relationship between the two variables. Feeding the off-diagonal information into the model should improve model performance.

$$\begin{bmatrix} a_{11} & a_{12}\dots & a_{1N} \\ a_{21} & a_{22}\dots & a_{2N} \\ a_{N} & a_{N2}\dots & a_{NN} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{21}\dots & a_{N1} \\ a_{12} & a_{22}\dots & a_{N2} \\ a_{1N} & a_{2N}\dots & a_{NN} \end{bmatrix} = \begin{bmatrix} cov(X,X) & cov(X,Y) & cov(X,Z) \\ cov(Y,X) & cov(Y,Y) & cov(Y,Z) \\ cov(Z,X) & cov(Z,Y) & cov(Z,Z) \end{bmatrix}$$

Step 5: Project the data onto new dimensions

3.2.2 Improving Signal Noise Ratio (SNR) via PCA

First, center the data. This is achieved by obtaining the z-scores. To do this, the explanatory variables are standardized and the average value is subtracted from the respective x_i's to convert each dimension into their respective z-scores. The centered data is then converted into the covariance matrix, on which the eigenfunction is applied.

The dimensions have now been transformed into a new set of dimensions by a rotation of axes in a mathematical space producing two new dimensions called eigenvectors E1 and E2. These eigenvectors are the principal components.

The eigenvectors are the directions in the original mathematical space where the maximum information content is captured. The amount of information each eigenvector captures (the variance across each eigenvector) is represented by the eigenvalues.

The eigenvectors are on the axes at 90 degrees to each other (I.e. orthogonal), and contain all the information content and data points. Each eigenvector is associated with an eigenvalue and the number of dimensions in the original space determines the number of principal components.The covariance matrix of the eigenvectors are constructed. The diagonal elements have a value of 1 and explain all the information in the data. The off-diagonal elements, with a value of zero, have no information content.

3.2.3 Algebraic Operations for PCA

PCA is a type of Singular Value Decomposition (SVD) which involves breaking down or decomposing a larger value into smaller values. The covariance matrix (the larger singular value) is decomposed, after scaling the independent variables, into smaller values and their eigenvalues obtained.

High dimensional data is difficult to analyze or visualize to identify hidden patterns. There is need to reduce the dimensionality so as to get better visualizations and results without losing most of the information.

The central idea behind PCA is to reduce the dimensionality of a dataset consisting of many variables that are correlated with each other while retaining the variation present in the data set. This is done by transforming the variables to a new set of orthogonal variables known as principal components. The first principal component retains the greatest amount of variation present in the original variables, followed by the second principal component and the third, etc. in decreasing order of magnitude.

3.2.4 SCALING

Input variables in a data set may have different units e.g. Km, hrs, kg, etc i.e. different scales.

Scaling the variables enables us to compare the variables on an equal level. One of the ways of scaling is normalization.

Normalization: This makes all the values to lie between 0 and 1.

$$y = {(x - \min)}/{(\max - \min)}$$

Here Y = Normalized variable; X = Variable of interest

Standardization: In this method of scaling, the mean of observations is 0 and the standard deviation is 1.

$$y = {(x - \text{mean})}/{\text{std\_dev}}$$

Where Y = standardized variable; X = variable of interest

Mean = average of variable x; Std_dev = standard deviation of x

4. RESULTS AND DISCUSSION

There are 1,683,462 data points and 21 columns in the data set. All columns have non-null values and are numeric except the date column which is of object data type. Because summary statistics show very high maximum figures for some variables (Foreign reserves position of 62,081.86) and rather low maximum figures for others (Savings rate of 4.28), there is need for scaling the data. For this, standardization and normalization can be used.

Table 1. Summary statistics

Mean	Std	min	25%	50%	75%	max	
167	38654.68515	8211.386882	23689.87	32952.845	37105.27	43382.795	62081.86
36	7.921111	4.988639	0	4.3875	7.665	11.7425	16.71
168	2.912083	0.984059	1.25	1.83	3.135	3.89	4.28
168	3.812024	1.493069	0.78	2.6775	4.055	4.66	7.32
168	7.648333	2.828773	1.64	6.1975	8.185	8.885	15.01
168	8.312798	2.616935	2.74	6.535	8.755	9.6825	14.65

168	8.440655	2.889185	2.65	5.91	8.63	10.625	15.84
168	8.275536	2.842299	3.53	5.64	8.015	10.575	16.47
168	8.46131	2.390413	2.17	6.925	8.6	9.8025	14.13
168	16.092679	2.029009	11.13	15.505	16.595	17.1025	19.66
168	25.960179	3.651563	17.58	23.195	26.07	28.74	31.56
120	231.691667	80.91535	130.19	159.3925	214.64	290.4225	411.52
120	219.209583	74.970089	121.94	153.4575	201.715	276.5575	385.93
167	5589.107066	2149.785464	1920.61	3843.425	5094	7472.71	10906.09
167	4473.393593	1234.995921	2502.6	3521.24	4460.7	5182.925	8574.64
167	4090.056587	1143.753746	2297.11	3185.865	4031.42	4760.115	7895.15
167	10062.50114	2907.098262	5027.94	7710.385	9798.31	12404.85	17318.66
167	1353.959042	1660.728018	-3480.64	0	1096.25	2367.97	5996.98

Table 1 continued

The distributions were checked for outliers. Distribution plots showed that Foreign reserve position and Crude oil price had outliers while Imports and Total trade were moderately right-skewed. Checking for correlations, we find high positive correlation between Crude oil price and the exchange rate (WDAS), as expected, Exports and WDAS and other expected positive correlations: low positive correlations between Exports and Time deposits. High negative correlation between Maximum lending rates and Exchange rates, as well between Foreign reserves position and Inflation; Low negative correlation between WDAS and Time deposits.

Step 1: Scale the Data: For scaling, we opted for standardization using StandardScaler.

Table 2. Scaled data

0	1	2	3	4	5	6	7	8	9	...
0	-0.909678	1.900764	0.56179	0.609844	0.857696	-0.032701	1.248121	1.138751	1.083579	0.721851
1	-0.91196	2.229664	0.564843	0.74827	1.786772	-0.083662	1.107051	1.10684	1.052917	0.704494
2	-0.919328	2.577554	0.560994	0.946835	0.727584	0.028454	1.778814	1.050109	0.79996	0.444129
3	-0.912272	2.70695	0.56033	1.409442	1.231766	0.059031	0.542769	1.18839	1.321205	0.940558
4	-0.908951	2.507151	0.559799	1.7596	-0.565397	-0.155009	0.717428	1.160025	1.248384	0.982216
...										
11	12	13	14	15	16	17	18	19	20	
...	0.838627	-0.283089	-2.115039	-1.259678	-1.302885	0.930885	-0.974044	-0.957979	0.274591	1.785471
...	0.89737	-0.25343	-2.142506	-1.25521	-1.287481	1.154541	-1.068278	-1.071578	0.399954	2.145067
...	0.515544	-0.426444	-2.301817	-1.229396	-1.271542	1.132113	-0.187075	-0.246295	0.757719	1.46073
...	1.44703	-0.347352	-2.117786	-1.227286	-1.254531	2.195067	-0.14334	-0.169098	1.562348	2.804183
...	1.333742	-0.530252	-2.191948	-1.214876	-1.237788	1.966083	0.146031	0.209891	1.515949	2.292577

Having scaled the data, we then applied Principal Component Analysis (PCA). Principal Component Analysis is a statistical procedure to convert observations of possibly correlated variables into new principal components such that these principal components:

1. are uncorrelated with each other

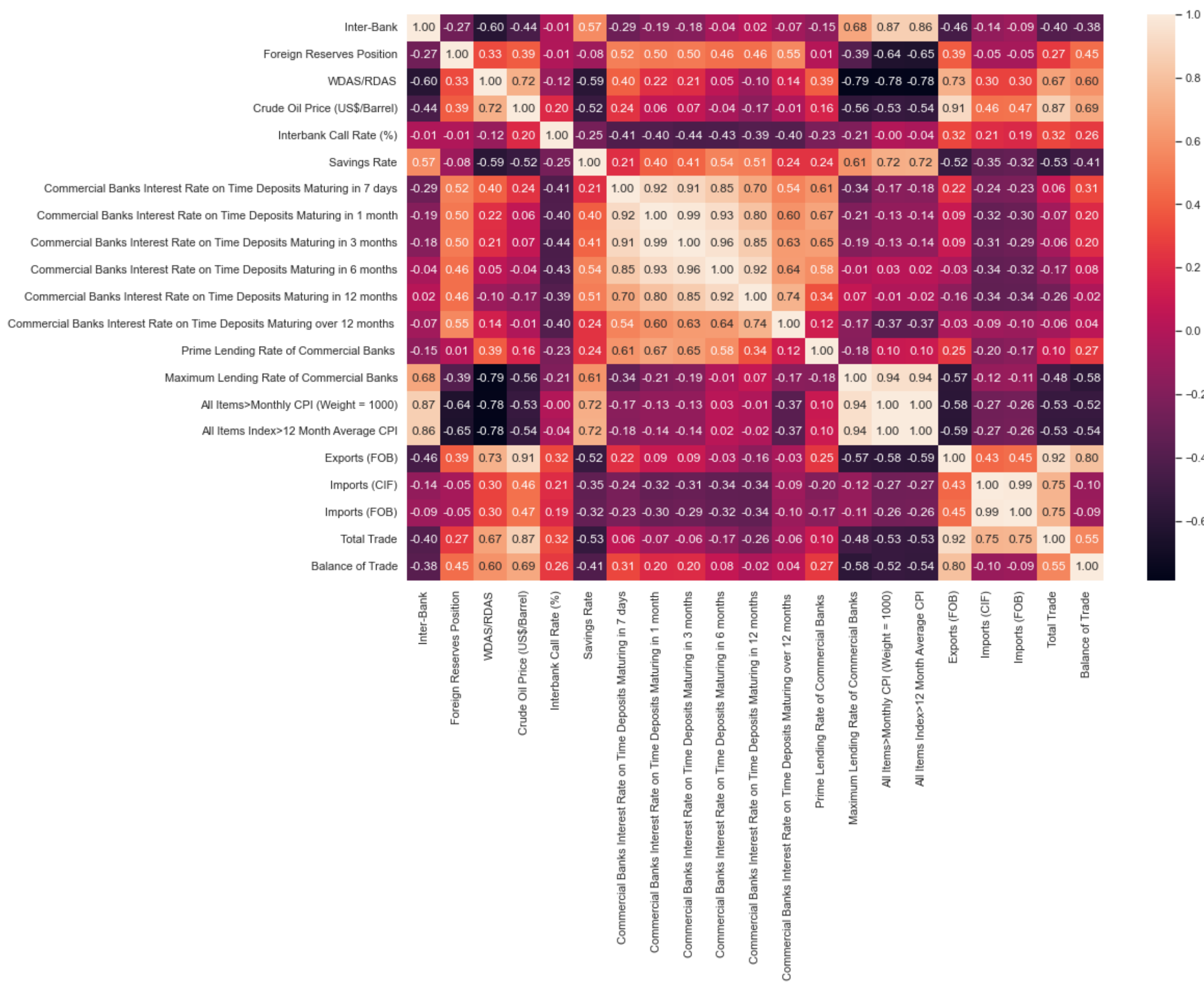
2. are linear combinations of the original variables

3. capture maximum information.

Step 2: Obtain covariance matrix

Covariance matrices were calculated after standardizing the data. Twelve non-null arrays were obtained as shown below.

Fig 1. Heat map



Step 3: Eigen Decomposition

The covariance matrix was then decomposed into Eigenvalues and eigenvectors. Eigenvectors do not change direction when a linear transformation is performed on them. They simply scale the vector. The value of the scaled vector is called the eigenvalue and is denoted by lambda in the equation: $AV=\lambda V$

Where : A is the transformation matrix; V is an eigenvector and λ is an eigenvalue

Table 3. Covariance matrix

array([[1.00598802, 0. , -0.60425382, -0.44680882, 0. ,
0.57735796, -0.29159561, -0.18934486, -0.18348724, -0.041429 ,
0.01955311, -0.07006487, -0.15079024, 0.68397935, 0. ,
0.],
[-0.60425382, 0. , 1.00598802, 0.7260656 , 0. ,
-0.59037684, 0.40194705, 0.22379434, 0.21155808, 0.05021323,
-0.09769131, 0.13669106, 0.38833292, -0.79214124, 0. ,
0.],
[-0.44680882, 0. , 0.7260656 , 1.00598802, 0. ,
-0.51975637, 0.23857122, 0.06219955, 0.07140967, -0.04141337,
-0.16670411, -0.01285407, 0.16164673, -0.56205364, 0. ,
0.],
[0.57735796, 0. , -0.59037684, -0.51975637, 0. ,
1.00598802, 0.21353976, 0.40178964, 0.41707847, 0.53997711,
0.51530856, 0.23979216, 0.2402474 , 0.60885901, 0. ,
[-0.29159561, 0. , 0.40194705, 0.23857122, 0. ,
0.21353976, 1.00598802, 0.92361744, 0.91184957, 0.85086453,
0.70333342, 0.54080883, 0.61081034, -0.34034546, 0. ,

0.],
[-0.18934486, 0. , 0.22379434, 0.06219955, 0. ,
0.40178964, 0.92361744, 1.00598802, 0.9911884 , 0.93995867,
0.809752 , 0.60216186, 0.67776113, -0.21581329, 0. ,
0.],
[-0.18348724, 0. , 0.21155808, 0.07140967, 0. ,
0.41707847, 0.91184957, 0.9911884 , 1.00598802, 0.96265395,
0.85162216, 0.63843588, 0.65139332, -0.19474609, 0. ,
0.],
[-0.041429 , 0. , 0.05021323, -0.04141337, 0. ,
0.53997711, 0.85086453, 0.93995867, 0.96265395, 1.00598802,
0.92089509, 0.64380966, 0.58291804, -0.00775366, 0. ,
0.],
[0.01955311, 0. , -0.09769131, -0.16670411, 0. ,
0.51530856, 0.70333342, 0.809752 , 0.85162216, 0.92089509,
1.00598802, 0.73981609, 0.3468831 , 0.07287045, 0. ,
0.],
[-0.07006487, 0. , 0.13669106, -0.01285407, 0. ,
0.23979216, 0.54080883, 0.60216186, 0.63843588, 0.64380966,
0.73981609, 1.00598802, 0.1211015 , -0.16953295, 0. ,
0.],
[-0.15079024, 0. , 0.38833292, 0.16164673, 0. ,
0.2402474 , 0.61081034, 0.67776113, 0.65139332, 0.58291804,
0.3468831 , 0.1211015 , 1.00598802, -0.17887031, 0. ,
0.],
[0.68397935, 0. , -0.79214124, -0.56205364, 0. ,
0.60885901, -0.34034546, -0.21581329, -0.19474609, -0.00775366,
0.07287045, -0.16953295, -0.17887031, 1.00598802, 0. ,

Each eigenvector has an eigenvalue. Eigenvalues determine the relative importance of the eigenvectors

Step 4: Sort in decreasing order of eigenvalues and use Explained Variance use to select the number of components.Twelve of the components had non-null values as shown in Table 5.

Table 4. Eigenvalues

array	([5.70681838, 3.61602839, 1.00256536, 0.59003164, 0.3916685 ,
	0.25394989, 0.1987761 , 0.13985333, 0.09354223, 0.04579133,
	0.00942869, 0.02340245, 0. , 0. , 0. ,
	0. , 0. , 0. , 0. , 0. ,
	0.])

Table 5. Explained variance

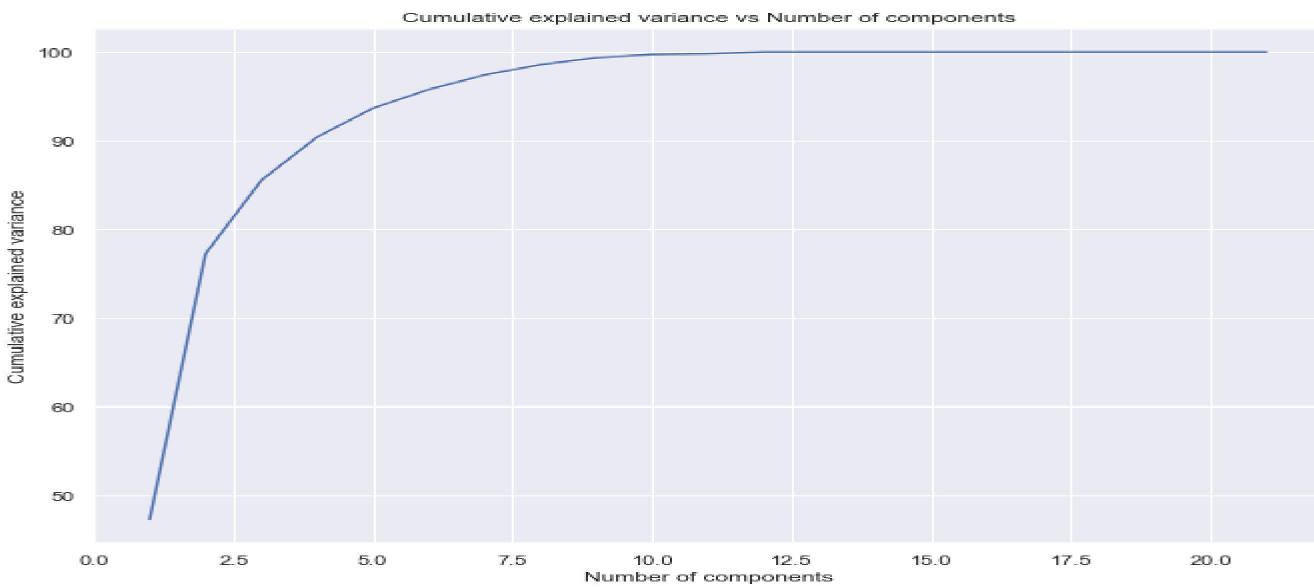
1	[47.27374353516036,		7	1.646607554141368,
2	29.95420344227172,		8	1.1585072268921324,
3	8.30498093444442,		9	0.7748785989010428,
4	4.887662850156739,		10	0.37932301735138196,
5	3.244476155859624,		11	0.07810468822742336,
6	2.103652379359654,		12	0.19385961723414977,

Table 6. Cumulative variance explained

array	([47.27374354, 77.22794698, 85.53292791, 90.42059076,
	93.66506692, 95.7687193 , 97.41532685, 98.57383408,
	99.34871268, 99.72803569, 99.80614038, 100. ,
	100. , 100. , 100. , 100. ,
	100. , 100. , 100. , 100. ,
	100.])

Using the Elbow method, the graph indicates that the first twelve components explain almost all the variance in the data. In fact the first five components are enough to explain about 95% variance in the data. So we can now project our data on five components instead of 21, thus reducing the dimension of the data. We could still go further to project on the first three components to achieve more than 70% variance. Explained Variance was used to select the number of components.

Fig 2. Cumulative explained variance



Number of PCs that explain at least 70 variance: 3

This provides the optimal number of explanatory variables to be used in modelling the determinants of Naira/US Dollar currency exchange rates. For even more robust outcomes, the graph indicates that 5 principal components explain more than 90 percent of the variance.

Step 5. Rearrange the original data on the final components and project the data into lower dimensions.

Fig 3. Scatterplot of new features

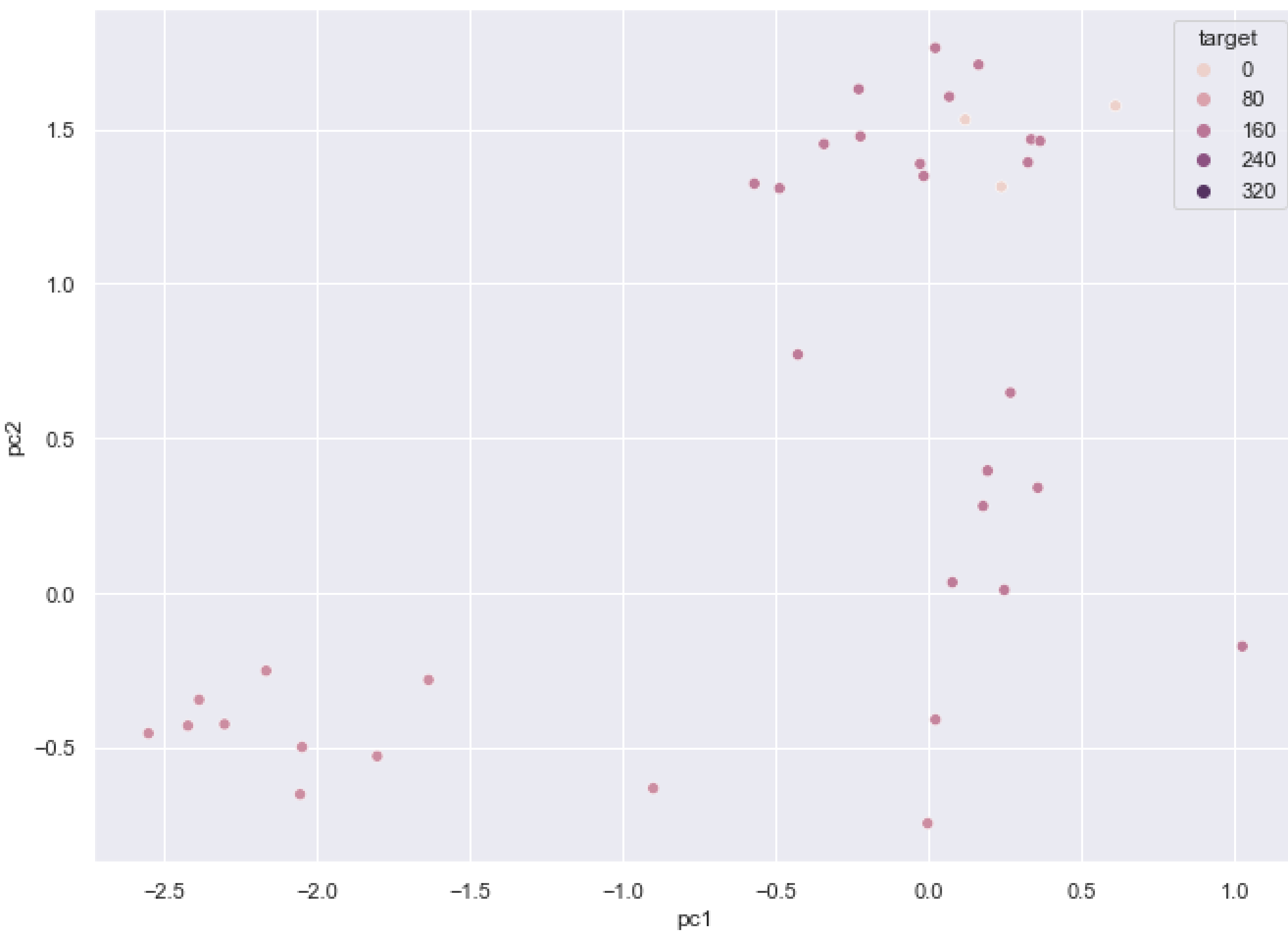


Table 7. Data projected onto lower dimensions

	Pc1	Pc2	Pc3	Pc4	Pc5			Pc1	Pc2	Pc3	Pc4	Pc5
0	-0.249	0.048	0.211	0.033	0.471		11	0.131	0.243	0.082	0.443	-0.028
1	0.243	0.13	-0.076	0.357	0.362		12	0.138	0.201	0.108	-0.488	-0.191
2	0.33	-0.073	-0.047	-0.141	-0.216		13	-0.305	0.058	0.26	-0.015	0.263
3	0.307	-0.136	0.118	-0.145	0.231		14	-0.236	0.062	0.154	-0.322	0.031
4	-0.004	-0.047	-0.026	-0.033	0.094		15	-0.238	0.062	0.156	-0.321	0.02
5	-0.187	0.271	0.26	-0.052	0.193		16	0.32	-0.133	0.099	-0.186	0.289
6	0.213	0.297	0.058	-0.113	-0.093		17	0.108	-0.242	0.506	0.153	-0.164
7	0 .167	0.348	0.082	-0.071	-0.087		18	0.109	-0.237	0.522	0.132	-0.135
8	0.167	0.351	0.101	-0.045	-0.064		19	0.283	-0.201	0.288	-0.073	0.144
9	0.109	0.37	0.148	-0.008	0.005		20	0.276	-0.028	-0.237	-0.228	0.471
10	0.064	0.357	0.11	0.183	0.034							

5. CONCLUSION

The determinants of currency exchange rate of the Naira/US Dollar was modeled in this study. Monthly data on some major determinants of exchange rates in the extant literature were obtained from Central Bank of Nigeria statistical database for the years 2008-2021. Because of the vary large number of factors indicated as determinants of Naira/US Dollar currency exchange rates, and also because many of these macro-economic variables are highly correlated and present the problem of multicollinearity, Principal Component Analysis (PCA) was applied to the data in order to eliminate the problems of multicollinearity, redundancy in explanatory variables and high dimensionality. Machine Learning tools and Python computer language were deployed in the analyses to drastically reduce the number of components from twenty-one to five. It was in fact found that the number of principal components that explain at least seventy percent of the variance was just three, while five principal components adequately explained more than ninety percent of the variance.The implication is that in future research, three major explanatory variables would be sufficient to analyse the determinants of currency exchange rates in Nigeria, and, if extra robustness is required, five principal components should be more than adequate.

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