PERFORMANCE ANALYSIS OF LISTED INSURANCE COMPANIES IN NIGERIA: A STOCHASTIC DOMINANCE APPROACH

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Abstract

This study undertakes a comparative performance analysis of listed insurance companies in Nigeria for the period 2005 – 2015, using stochastic dominance analysis. For all intents and purposes, the stochastic dominance analysis of the twenty-four (24) listed insurance firms in Nigeria was carried out based on their profit or loss for the period of the study. Also, the top ten (10) insurance companies with the highest performance rate, complemented by the 11-year average performance, were selected for further comparative assessments. The comparative assesment results show that, Lasaco Assurance, Sovereign Trust Insurance, Consolidated Hallmark Insurance, Axamansard Insurance, Nem Insurance, Prestige Assurance, Staco Insurance, Aiico Insurance, Mutual Benefits Assurance and Regency Assurance emerged the top ten (10) performing insurance companies in Nigeria for the period of study. The findings from the stochastic dominance analysis revealed inter alia that, Axamansard Insurance stochastically dominates the Nigerian insurance market. This is closely followed by Lasaco Assurance, NEM Insurance as well as Sovereign Trust Insurance, Prestige Assurance and Aiico Insurance respectively. The empirical findings of this study suggest three likely circumstances that can drive the decision of any rational economic agent. First, individual or organization will tend to go for the insurance firm(s) that first-order stochastically dominate(s) the most, given all conceivable pair comparison. Second, the individual or organisation may be a risk-averse entity, in this regard, such individual or organization will go for the firm or firms that second-order stochastically dominate (s) the most. Finally, there are situations that neither makes the individual or organization better-off, nor worse-off. In this regard, the individual or organization will remain indifferent. This occurs when the outcomes returned inconclusive. In this respect, the individual or organization can choose either of the pair with no relative advantage.

Keywords: Stochastic Dominance, Risk Averse, Utility Functions, Comparative Risk, Nigeria.

1.0 Introduction

The problem of comparing and ordering various random outcomes represents a huge challenge in theoretical and applied research that occurs in numerous instances in decision. Financial decision making is no exception. Indeed, an investor has to allocate her wealth among available assets based on their joint distribution over all possible states of nature making (Jianwei & Feng, 2017). If full information concerning the distribution of underlying variables of interest is available, it is natural to use this full information in the decision-making process, rather than only certain characteristics of that distribution, such as its mean or variance suggested by the classical mean-variance (MV) context by Markowitz (1952).

For many years economists thought that mean and variance are satisfactory measure of comparative risk. But this was not quite right. Indeed, mean and variance can only serve this purpose if agents' utility functions are quadratic or if all probability distribution are normal distributions (Wong, 2007). Surely, normal distributions are too restrictive, and quadratic utility functions are unsatisfactory. Not only do they imply that utility reaches a maximum; they also entails that the absolute degree of risk aversion is increasing in wealth, approaching infinity as utility approach its maximum. Consequently, one is lead to the absurd result that the willingness to gamble for a bet of fixed size should decrease as wealth is increased. Also, the MV does not accomplish the monotonicity assumption (regarding to first-degree stochastic dominance) and are restricted to the normal games order (Wolfstetter, 1996).

Another measure, stochastic dominance (SD), as partial orders defined over a set of risky payoffs, also provides useful criterion for portfolio choice and risk management. Stochastic Dominance (SD) relation is a decision-making rule which uses full information for the ordering of uncertain prospects. Unlike parametric criteria such as Mean-Variance analysis, SD accounts for the whole range of distribution function, rather than its particular characteristics such as first two moments, (i.e., mean and variance) like traditional financial indexes. Nevertheless, the practical use of this framework in financial analysis both academic and industrial has been very limited, due probably to the lack of simplicity of its interpretations and also the complexities of calculations (Wolfstetter, 1996).

The idea of SD was first mooted by Daniel Bernoulli (1738), when he suggested the comparison of random outcomes by converting them into their utilities, before computing for the expectations. Modern application of the concept in mathematics, finance and economics was introduced by Mann and Whitney (1947), Lehmann (1955) and Quirk & Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970) respectively. The SD approach has been regarded as one of the most useful tools to rank investment prospects when there are uncertainties (see, for example, Levy 1992) as the ranking of the assets has been proven to be equivalent to utility maximization for the preferences of risk averters and risk lovers (see, for example, Quirk & Saposnik, 1962; Hanoch & Levy, 1969; Hammond, 1974; Stoyan, 1983; Li & Wong, 1999).

Since SD rules have been demonstrated to offer, in many cases, superior and more efficient criteria on which to base investment decisions than the criterion derived from the traditional model of asset choice based on MV methodology. The use of SD theory to compare profit or return for risk averters has been well established in both theory and application. Theoretical works linking the SD theory to the selection rules for risk averters and risk lovers under different restrictions on the utility functions has also been well investigated (for example, Quirk & Saposnik, 1962 and Hammond, 1974).

Stochastic dominance (SD) is one of the most famous approaches to comparing pairs of prospects. Well-known specifications of SD are first order SD (FSD) and second order SD

(SSD), which by far attract most of the attention in SD research. The first and second order stochastic dominance indicates when one random variable ranks higher than the other by specifying a condition which the difference between their distribution function must satisfy. Essentially, first order stochastic dominance is a "stochastically larger", and second order stochastic dominance a "stochastically less volatile" or "less risky" relationship; while the "larger" random variable is preferred by all agents who prefer higher realizations, the "less volatile" random variable is preferred by all agents who also dislike risk. In this sense, stochastic dominance theory provides unanimity rules, provided that utility functions have certain common properties (Heyer, 2001).

Due to the advantage mentioned above, the SD approach has been proved to be a powerful tool for ranking random variables and employed in various areas of finance, decision analysis, economics and statistics (See, Meyer, 1989; Levy, 1992, 2006; Chiu, 2005; Li, 2009; Blavaskyy, 2010, 2011; Deutsch and Silber, 2011; Bibi, Duclos and Audrey, 2012; Yalonetzky, 2012; Tzeng, Huang & Shih, 2013; Loomes, Inmaculada & Pinto-Prades, 2014; Valentini, 2015;Tsetlin, Winkler, Huang & Tzeng, 2015).

To the best of the authors' knowledge, there are few studies utilizing the SD criteria to compare or rank the performance of listed insurance companies in Nigeria. This study attempts to fill this gap. Hence, the main objective of this study is to carry out a comparative performance analysis of listed insurance companies in Nigeria, using stochastic dominance approach.

The remaining part of the paper is structure as follows. Section two reviews the literature relating to the study. Section three describes the methodology used in the empirical study. Section 4 shows the results regarding the rankings derived under the stochastic dominance approach. Finally, section 5 concludes the paper.

2.0 Literature Review

2.1 Conceptual Literature

Properties of Utility Functions

As stochastic dominance is a generalisation of utility theory, we will begin the conceptual review with a discussion of utility functions. Simply stated, a utility function measures the relative value that a firm places on a business outcome. Within this definition, however, lies a significant limitation of utility theory: we can compare competing options, but we cannot assess the overall acceptability of any of those options. In other words, there is not objective, absolute scale for utility.

To specify a utility function we must have a measure that uniquely identifies each business outcome, typically some measure of profitability or terminal wealth, and a function that maps each business outcome to its corresponding utility. By convention utility is purely an ordinal measure. In other words, utility can be used to establish the rank ordering of outcomes, but cannot be used to determine the degree to which one is preferred over the other. For example, consider two outcomes ***A*** and ***B*** with corresponding utilities of 100 and 25. We can say that ***A*** is preferred over ***B***, but we cannot say that ***A*** is four times more preferred than ***B***. As a consequence of this ordinality, utility functions are not unique. Any positive, linear transformation of a utility function will still yield the same rank ordering of investment alternatives (Heyer, 2001).

Unfortunately, we rarely know *a priori* what outcomes will result from various investment alternatives. Instead, forecasted terminal wealth has some distribution which varies depending upon the investment alternative selected. Classical utility theory assumes that rational firms seek to maximize their expected utility and choose among their investment alternatives accordingly. Mathematically, this is expressed as:

A is preferred to B if and only if terminal wealth satisfies $E_w[U(w_A)] - E_w[U(w_B)] \geq 0$ with at least one strict inequality $U(w_A) - U(w_B) \geq 0$ ……………………………. (1)

The mathematical features of the utility function U reflect the risk/reward motivations of the firm: several common risk/reward features are discussed below. These same features also determine what stochastic characteristics the terminal wealth distribution must possess if one alternative is to be preferred over another. Evaluation of these stochastic characteristics is the basis of stochastic dominance analysis.

Increasing Wealth Preference

This feature captures the "more wealth is better" philosophy of firm behavior and is generally considered a universal feature of utility functions. For greater wealth to be preferred, the utility function must be monotonically increasing. Mathematically this is expressed as:

A utility function possesses increasing wealth preference if and only if $U'(w) \geq 0$ *for all w with at least one strict inequality. ……………………………………………………… (2.1)*

Risk Aversion

This feature captures the willingness of a firm to purchase insurance *(i.e.,* to pay more than the expected loss to transfer an insurable loss). This is a subset of increasing wealth preference; a firm may have increasing wealth preference with or without exhibiting risk aversion, and is also generally considered a universal feature of utility functions. Mathematically this is expressed as:

A utility function possesses risk aversion if and only if it satisfies the conditions for increasing wealth preference and $U'(w) \leq 0$ *for all w with at least one strict inequality……………………………………………………………………. (2.2)*

It is not intuitively clear, however, that this mathematical definition of risk aversion is equivalent to the behavioral definition given above. To make this relationship clearer we must recognize that Equation *2.2* defines a concave function and apply Jensen's inequality. This yield:

$$E_w[U(w)] \leq U(E_w[w])$$

Under risk aversion, then, the expected utility of a risky investment is less than the utility of the expected outcome. Why should this be the case? By proposition the firm has penalized the utility of the investment for the possibility of unfavorable outcomes. If we rewrite Jensen's inequality with a strict inequality we can show that:

$$E_w[U(w)] \leq U(E_w[w]k)$$

This shows that the firm is indifferent between the return on a risky investment or a lower, risk-free wealth equal to $E_w[w]k$ where k is the premium that the firm is willing to pay to eliminate risk (Heyer, 2001).

Skewness Preference (Ruin Aversion)

This feature is classically presented as an individual's willingness to play the lottery: to accept a small, almost certain loss in exchange for the remote possibility of huge returns. A firm's concern, however, is with the opposite situation, unwillingness to accept small, almost certain gain in exchange for the remote possibility of ruin. This is a subset of risk aversion; a firm may have risk aversion with or without exhibiting ruin aversion. Mathematically this is expressed as:

A utility function possesses ruin aversion ff and only if it satisfies the conditions for risk aversion and $U''(w) \geq 0$ for all w with at least one strict inequality…………………… (2.3)

As with risk aversion, it is not intuitively clear that the mathematical and behavioral definitions of ruin aversion are consistent. If we take a Taylor series expansion of the utility function about $E_w[w]$, and take the expectation with respect to w, we obtain:

$$U(w) = U(E_w[w]) + U'(E_w[w]) \cdot (w - E_w[w]) + \frac{U''(E_w[w])}{2!} \cdot (w - E_w[w])^2 + \frac{U''(E_w[w])}{3!} \cdot (w - E_w[w])^3$$

$$E_w[U(w)] = U(E_w[w]) + \frac{U''(E_w[w])}{2!} \cdot \sigma_w^2 + \frac{U''(E_w[w])}{3!} \cdot \mu_3$$

From this expression we can see that any investment feature that increases positive skewness μ_3 (or reduces negative skewness) acts to increase expected utility (Heyer, 2001).

Stochastic Dominance

Stochastic Dominance (SD) is a probabilistic concept of relation among different random variables. According to Lizyayev (2010), SD relation is a decision making rule which uses limited information for the categorising of uncertain prospects. Stochastic dominance is a generalization of utility theory that eliminates the need to explicitly specify a firm's utility function. Rather, general mathematical statements about wealth preference, risk aversion, etc. are used to develop optimal decision rules for selecting between investment alternatives. Presented in the context of expected utility theory, the SD approach has the advantage that it requires no restrictions on probability distributions.

Utility theory and the features embedded in utility functions are elegant but practically ineffective constructs. An economist once asserted: *"A man who seeks advice about his actions will not be grateful for the suggestion that he maximize expected utility."* Few firms have the willingness or means to select and parameterize their own utility function, The problem becomes, then, how can we use features such as increasing wealth preference, risk aversion, ruin aversion, etc. to select among investment alternatives without actually selecting a specific utility function?

First-Order Stochastic Dominance

Let us begin with the definition of preference given in Equation 1 and the most general constraint on a utility function given in Equation 2.1, increasing wealth preference. We can integrate Equation 1 by parts to yield:

$$E_w[U(w_A)] - E_w[U(w_B)] \geq 0$$

$$\int_{\infty}^{\infty} U(t)\int_{A}(t)dt - \int_{\infty}^{\infty} U(t)\int_{B}(t)dt \geq 0$$

$$\int_{\infty}^{\infty} U(t) \cdot \left[\int_{A}(t) - \int_{B}(t)\right] dt \geq 0$$

$$U(t) \cdot [F_A(t) - F_B(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} [F_A(t) - F_B(t)] \cdot U'(t)dt \geq 0$$

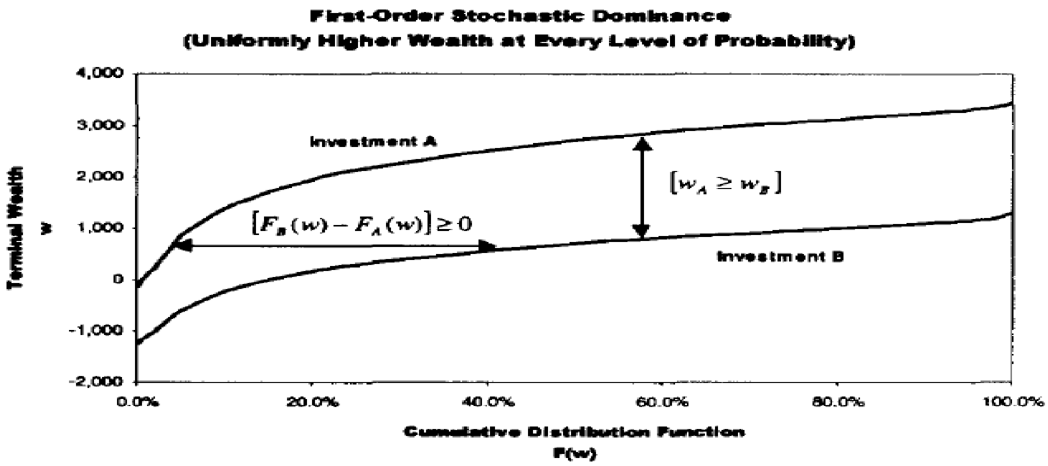
$$\int_{\infty}^{\infty} [F_B(t) - F_A(t)] \cdot U'(t)dt \geq 0 \; \ldots\ldots\ldots.. \qquad 3.1$$

By Equation 2.1 we know that $U'(w) \geq 0$ so for Equation 3.1 to be true for all utility functions with increasing wealth preference we must have:

***A** is uniformly preferred to **B** under increasing wealth preference (A dominates B by first-order stochastic dominance) if and only if $[F_B(w) - F_A(w)] \geq 0$ for all w with at least one strict inequality. ……………………………………………………………..* (3.2)

Practical understanding of this constraint is straightforward if we place it on a Lee-graph. This is shown in Figure 1 below. Note that this graph depicts ultimate wealth rather than ultimate loss as is commonly shown in actuarial applications.

Figure 1



Source: Heyer, D. D. (2001). Stochastic dominance: a tool for evaluating reinsurance alternatives, in: Casualty Actuarial Society Forum, p 100.

This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 3.2. From this graph we can see that first-order stochastic dominance is equivalent to uniformly higher terminal wealth at every level of probability. Accordingly, first-order stochastic dominance is a weak result; rarely will a firm be faced with such an obvious investment choice. The weakness of this result arises from the fact that first-order stochastic dominance results from a weak utility function constraint, increasing wealth preference.

Second-Order Stochastic Dominance

Let us now use a stronger utility function constraint, risk aversion, to develop investment selection criteria. We begin with the definition of preference given in Equation 1 and the risk aversion definition given in Equation *2.2.*

We can twice integrate Equation 1 by parts to yield:

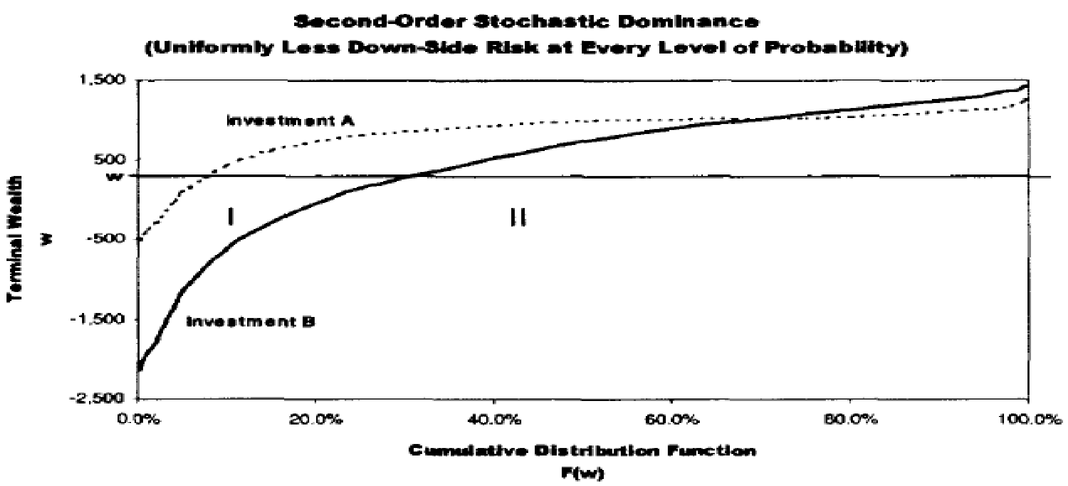
$$U'(\infty)\cdot \int_{-\infty}^{\infty}[F_B(t)-F_A(t)dt]-\int_{-\infty}^{\infty}U''(t)\int_{-\infty}^{\infty}[F_B(u)-F_A(u)]dudt\geq 0 \qquad (4.1)$$

Since risk aversion is a subset of increasing wealth preference we know that $U'(\infty)\cdot\int_{-\infty}^{\infty}[F_B(t)-F_A(t)dt]$is positive. By Equation 2.2 we know that $U''(w)\leq 0$ so for Equation 4.1 to be true for all utility functions with risk aversion we must have:

***A** is uniformly preferred to **B** under risk aversion (A dominates B by second-order **stochastic** dominance) if and only if* $\int_{-\infty}^{\infty}[F_B(u)-F_A(u)]du\geq 0$ *for all w with at least one strict inequality.* ……………………………………………………………………………… (4.2)

Again, practical understanding of this constraint is straightforward if we place it on a Lee-graph. This is shown in Figure 2 below.

Figure 2



Source: Heyer, D. D. (2001). Stochastic dominance: a tool for evaluating reinsurance alternatives, in: Casualty Actuarial Society Forum, p 101.

This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 4.2. From this graph, it is obvious that first-order stochastic dominance does not apply in this case. The two cumulative distribution functions intersect and, consequently, neither investment option results in uniformly higher wealth at every level of probability. How, then, can we recognize second-order stochastic dominance? On a Lee-graph, the limited expected value of investment A (limited to wealth w) is depicted by areas I and II combined. Similarly, the limited expected value of investment B is depicted by area II. Area I, then, may be interpreted as the difference between the limited expected values of investments A and B. Area I is also the constraint integral in Equation 4.2 for a specific wealth *w. accordingly,* *second-order stochastic dominance is equivalent to a uniformly higher limited expected value at every wealth limit.*

By changing the variable of integration, it can also be shown that second-order stochastic dominance implies that area I is positive for every level of probability. This may be interpreted as "uniformly less down-side risk at every level of probability".

Third-Order Stochastic Dominance

Finally, let us use the definition of preference given in Equation 1 and the ruin aversion definition given in Equation 2.3. We can thrice integrate Equation 1 by parts to yield:

$$U'(\infty)\cdot\int_{-\infty}^{\infty}[F_B(t)-F_A(t)]dt-U''(x)\int_{-\infty}^{x}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\;\Big|_{\infty}^{\infty}\cdots$$

$$\cdots+\int_{-\infty}^{\infty}U''(w)\int_{-\infty}^{w}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\,dw\geq 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\quad (5.1)$$

Since risk aversion is a subset of ruin aversion we know that:

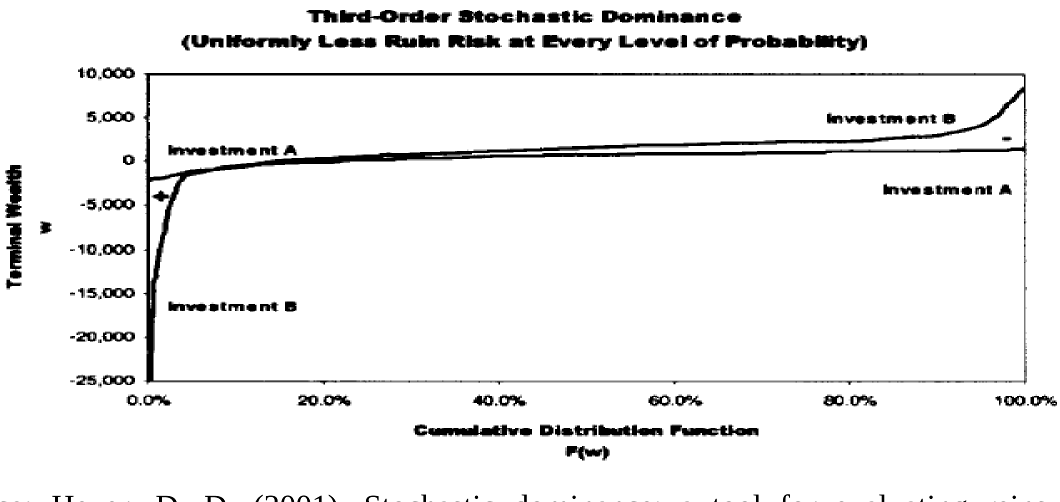
$$U'(\infty)\cdot\int_{-\infty}^{\infty}[F_B(t)-F_A(t)]dt-U''(x)\int_{-\infty}^{x}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\;\Big|_{\infty}^{\infty}\cdots$$

:is positive. By Equation 2.3 we know that $U''(w)\geq 0$, so for Equation 5.1 to be true for all utility functions with ruin aversion we must have:

A is uniformly preferred to B under ruin aversion (A dominates B by third-order stochastic dominance) if and only if $\int_{-\infty}^{w}\int_{-\infty}^{t}[F_B(u)-F_A(u)]dudt\geq 0$ *for all w with at least one strict inequality*……………………………………………………………….. (5.2)

This is shown graphically in Figure 3 below,

Figure 3



Source: Heyer, D. D. (2001). Stochastic dominance: a tool for evaluating reinsurance alternatives, in: Casualty Actuarial Society Forum, p 102.

This figure depicts the cumulative distribution functions for two investments A and B that satisfy Equation 5.2. From this graph, it is obvious that the cumulative distribution functions intersect and first-order stochastic dominance does not apply in this case. Similarly, although not readily apparent from the graph, the negative area between the cumulative distribution functions is 50% larger than the positive area so second order dominance does not apply in this case. Investment A, however, has significantly less negative skewness (remote, but possible ruin). Unfortunately, there is no simple graphical means to explicitly test whether investment A and B satisfy the conditions of Equation 5.2.

2.2 Theoretical Literature

Utility Theory

The expected utility (EU) hypothesis is the most popular approach to the difficulty of decision making under uncertainty. The theory posits that a risk averse investor needs to define his preference (utility function) to distinguish the best portfolio among an alternative set of choices (Leili, 2013). In general, the utility maximisation approach makes use of full available information of investor's preference. However, because only partial information of an investor's utility function is readily available, one must look for an approach that can utilise this partial information to select optimal investment portfolios and that is why this study focus on stochastic dominance which utilises partial (limited) information for decision making.

Traditional Portfolio Theory

The traditional theory of portfolio is based on the fact that risk could be measured on each asset through the process of computing the risk and that assets with the lowest risks should be chosen. The theory has been of a very subjective nature but it has provided success to some persons who have made their investments by making analysis of individual assets through evaluation of return and risk conditions (Francisco, 2007).

Modern Portfolio Theory (MPT)

The MPT explains how risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasising that risk is an inherent part of higher reward (Markowitz, 1952). The theory was developed by Harry Markowitz in his paper "portfolio selection" published in 1952 by the journal of finance, which explains the four basic steps involved in portfolio construction as security evaluation, asset allocation, portfolio optimization and performance management. The essence of coming up with the theory is to validate construction of an efficient portfolio through diversification to reduce risk by having investors combining assets from different industries and sectors. The assumptions of the MPT are;

Investors are rational and behave in a manner as to maximise their utility, investors have free access to information, market efficiency, investors are risk averse, investors base their decisions on expected returns and variance and investors prefer higher returns to lower returns for a given level of risk.

2.3 Empirical Literature

Lizyayev (2010) applied the concept of Stochastic Dominance for constructing a multi-period asset allocation strategy and analyzed its out-of-sample performance relative to other popular strategies, such as full- and lower-partial moment's strategies, and value and momentum rebalancing. The researcher find a relatively good performance of the SSD strategy

Leili (2013) studies portfolio selection by second order stochastic dominance based on the risk aversion degree of investors by developing a new efficiency model, SSD-DP, based on the linear programming technique and finds an SSD efficient portfolio by minimising the dual power transform of a weighted portfolio of assets for a given risk aversion degree i.e it is not dominated by SSD by any other portfolio.

Post and Kopa (2015) studies portfolio choice based on third-degree stochastic dominance, with an application to industry momentum by developing a portfolio optimization method for building investments portfolios that dominate a given benchmark index in terms of third-degree stochastic dominance and applying a problem reduction method based on vertex enumeration. The study reveals that relative to the benchmark, the constructed portfolio increases average out-of-sample return by almost seven percentage points per annum without incurring more downside risk.

Osifo (2018) utilize stochastic dominance analysis to determine an optimal investment portfolio for a given set of assets in Nigeria, using the Vose model risk software for the analysis. The result of the study reveals that earnings per share (EPS) is first order dominance over dividend per share (DPS), return on equity (ROE) and return on capital employed (ROCE).

3.0 Methodology

The data used in this study are secondary data which were sourced from the audited financial statements of the sampled insurance firms used in the study and the fact books of the Nigerian Stock Exchange. The data collected is the profit and loss for all the twenty-four (24) listed insurance firms. The study covers 2005 to 2015. The Vose Model Risk software was used for the analysis. The Model Risk provides a function Vose Dominance that produces a matrix of first and second order stochastic dominance results for a set of generated outputs.

4.0 Analysis of Results and Discussion

This section borders on the analysis of results and discussion of the empirical findings of the study. It also unveils the inferences for relevant policy actions. It is worth recalling that, the principal aim of this study is to critically carry out a comparative performance analysis of listed insurance companies in Nigeria, using stochastic dominance analysis. For all intents and purposes, twenty-four (24) listed insurance firms in Nigeria were accounted for, owing to data availability on the relevant phenomenon of interest (Profit/Loss). Thus, all empirical assessments were restricted to these variables of interest.

Specifically, the reported insurance companies include; Lasaco Assurance, Sovereign Trust Insurance, Consolidated Hallmark Insurance, Axamansard Insurance, Nem Insurance, Prestige Assurance, Staco Insurance, Aiico Insurance, Mutual Benefits Assurance, Regency Assurance, Law Union And Rock Insurance, Universal Insurance Company, Niger Insurance, Veritas Kapital Assurance, Wapic Insurance, Cornerstone Insurance, Linkage Assurance, Guinea Insurance, Standard Alliance Insurance, International Energy Insurance, Goldlink Insurance, Sunu Assurances Nigeria, Royal Exchange and African Alliance Insurance.

In addition, in achieving the above-mentioned purpose, the study employed Stochastic Dominance Analysis. Essentially, Stochastic Dominance Analysis is an analytical procedure for assembling shared preferences in a set of probable outcomes, in addition to their resulting occurrence probabilities. For instance, assuming a pool of observations on a particular outcome (profit/loss) for Firm X and Firm Y, Firm X is adjudged to be first-order stochastically dominant over Firm Y, if and only if for every performance outcome, Firm X is chosen in place of Firm Y.

A true value maximizing individual or organisation intending to take an insurance cover, may prefer insurance firm X to Firm Y, based on track record of those insurance firms. Thus, Firm X can be preferred to Y based on ease of performance predictability. Nevertheless, Firm X is said to be second-order stochastically dominate Firm Y, if and only if the policy dynamics associated with Firm X are more easily predicted than those associated with Firm Y. Essentially, policy dynamics in this regards border on *the modus operadii* of the insurance company in question, the stringent nature of the insurance policies operated by the company and the extent to which these policies apply to existing and prospective customers.

It thus suggests that, an individual may choose to take a cover in a particular insurance company with higher record of performance without considering the stringent nature of the policy governing such insurance cover (a risk-tolerance/risk-taking situation). Thus, in a bid to take a cover against one risk (accident for instance), one may still have to consider the complexities associated with such policy (such as motor vehicle insurance). An individual or organisation may choose to take a particular insurance cover in company X (based on performance trends) without considering the consequences of violating the underlying insurance policy. This is an instance of first-order stochastic dominance.

In the event of second-order stochastic assessments, the individual or organisation is more risk-aversive as such entity is much more concerned about the conditionalities of the relevant insurance policy, in addition to the underlying insurance cover. Thus, first-order stochastic dominance is a sufficient condition for second-order stochastic dominance, as they both revolve around prioritizing the predictability of a set of outcomes. In effect, the various results gotten in the course of assessment were summarized in tables and further deliberated in sequence. Finally, the last segment of this section presents the dominance analysis of the ten best performing insurance companies in Nigeria based on their profit and loss from 2005 to 2015.

4.1 Stochastic Dominance Analysis of the Listed Insurance Companies in Nigeria

This section presents the stochastic dominance analysis of the listed insurance companies in Nigeria. Stochastic dominance analysis is a diagnostic procedure for ranking mutual predictions, as they relate to a set of possible outcomes in addition to their equivalent manifestation likelihoods. Assuming in the Nigeria insurance market, there is a pool of observations on a certain outcome (e.g., financial performance) for insurance firm A and B. Insurance firm A is said to be first-order stochastically dominant over B, if and only if for every outcome, insurance firm A is preferred to insurance firm B.

A utility player, such as investor, may prefer insurance firm A to insurance firm B, on the basis of the understanding of the historical trends of these insurance firm. However, since the business environment is characterized by diverse type of risks, insurance firm A is said to second-order stochastically dominate insurance firm B, if and only if the risks related to insurance firm A are more easily predictable than those connected with insurance firm B. The implication of this is that, investors who want to invest in these insurance firms will prefer the firm that presents a higher return, notwithstanding the allied risks (a risk-tolerance/risk-taking investor). This is an instance of first-order stochastic dominance. However, in an event of second-order stochastic trends, the investor is risk-aversive/risk-intolerant. Thus, the former is a significant condition for the latter. To that effect, the result of the dominance analysis for the selected insurance companies, with emphasis on their performance, measure with profit or loss is shown in Table 4.1A through Table 4.1C.

From Table 4.1A, Aiico Insurance is first-order stochastically dominant over African Alliance Insurance, while Axamansard Insurance is first-order stochastically dominant over African Alliance Insurance and second-order stochastically dominant over Aiico Insurance. In addition, Guinea Insurance is second-order stochastically dominant over Cornerstone Insurance, Goldlink Insurance and International Energy Insurance, while International Energy Insurance is second-order stochastically dominant over Goldlink Insurance.

In similar fashion, the result from Table 4.1B reveals that Lasaco Assurance is first-order stochastically dominant over Law Union and Rock Insurance and Linkage Assurance, while it (Lasaco Assurance) is second-order stochastically dominant over Mutual Benefits Assurance. Likewise, Nem Insurance is second-order stochastically dominant over Niger Insurance and Regency Assurance, while Prestige Assurance is second-order stochastically dominant over Niger Insurance.

Furthermore, the result in Table 4.1C reveals that Sovereign Trust Insurance is first-order stochastically dominant over Royal Exchange, and second-order stochastically dominant over Staco Insurance and Standard Alliance Insurance, while Staco Insurance is first-order stochastically dominant over Royal Exchange and second-order stochastically dominant over Standard Alliance Insurance. However, Royal Exchange is second-order stochastically dominant over Standard Alliance Insurance, while Universal Insurance Company is first-order stochastically dominant over SUNU Assurances Nigeria and second-order stochastically dominant over Wapic Insurance. Similarly, Veritas Kapital Assurance is second-order stochastically dominant over SUNU Assurances Nigeria and Wapic Insurance, while Wapic Insurance is first-order stochastically dominant over SUNU Assurances Nigeria.

The result however returned inconclusive in the case of Consolidated Hallmark Insurance and African Alliance Insurance, Consolidated Hallmark Insurance and Aiico Insurance, Consolidated Hallmark Insurance and Axamansard Insurance, International Energy Insurance and Cornerstone Insurance, Goldlink Insurance and Cornerstone Insurance. The results further returned inconclusive in the case of Mutual Benefits Assurance and Law Union and Rock Insurance, Mutual Benefits Assurance and Linkage Assurance, Regency Assurance and Niger Insurance, Regency Assurance and Prestige Assurance, as well as Veritas Kapital Assurance and Universal Insurance Company. The results are reported in Table 4A through Table 4C respectively.

Table 4.1A: Dominance Analysis of Insurance Companies by Financial Performance (Profit and Loss)

Dominance	***African Alliance Insurance***	***Aiico Insurance***	***Axamansard Insurance***	***Consolidated Hallmark Insurance***
African Alliance Insurance		*Aiico Insurance Is 1d Over African Alliance Insurance*	*Axamansard Insurance Is 1d Over African Alliance Insurance*	*Inconclusive*
Aiico Insurance	*Aiico Insurance Is 1d Over African Alliance Insurance*		*Axamansard Insurance Is 2d Over Aiico Insurance*	*Inconclusive*
Axamansard Insurance	*Axamansard Insurance Is 1d Over African Alliance Insurance*	*Axamansard Insurance Is 2d Over Aiico Insurance*		*Inconclusive*
Consolidated Hallmark Insurance	*Inconclusive*	*Inconclusive*	*Inconclusive*	
Dominance	***Cornerstone Insurance***	***Goldlink Insurance***	***Guinea Insurance***	***International Energy Insurance***
Cornerstone Insurance		*Inconclusive*	*Guinea Insurance Is 2d Over Cornerstone Insurance*	*Inconclusive*
Goldlink Insurance	*Inconclusive*		*Guinea Insurance Is 2d Over Goldlink Insurance*	*International Energy Insurance Is 2d Over Goldlink Insurance*
Guinea Insurance	*Guinea Insurance Is 2d Over Cornerstone Insurance*	*Guinea Insurance Is 2d Over Goldlink Insurance*		*Guinea Insurance Is 2d Over International Energy Insurance*
International Energy Insurance	*Inconclusive*	*International Energy Insurance Is 2d Over Goldlink Insurance*	*Guinea Insurance Is 2d Over International Energy Insurance*	

Source: Author's computation using Vose Software (ModelRisk, 2020).

Table 4.1B: Dominance Analysis of Insurance Companies by Financial Performance cont'd

Dominance	***Lasaco Assurance***	***Law Union And Rock Insurance***	***Linkage Assurance***	***Mutual Benefits Assurance***
Lasaco Assurance		*Lasaco Assurance Is 1d Over Law Union And Rock Insurance*	*Lasaco Assurance Is 1d Over Linkage Assurance*	*Lasaco Assurance Is 2d Over Mutual Benefits Assurance*
Law Union And Rock Insurance	*Lasaco Assurance Is 1d Over Law Union And Rock Insurance*		*Inconclusive*	*Inconclusive*
Linkage Assurance	*Lasaco Assurance Is 1d Over Linkage Assurance*	*Inconclusive*		*Inconclusive*
Mutual Benefits Assurance	*Lasaco Assurance Is 2d Over Mutual Benefits Assurance*	*Inconclusive*	*Inconclusive*	
Dominance	***Nem Insurance***	***Niger Insurance***	***Prestige Assurance***	***Regency Assurance***
Nem Insurance		*Nem Insurance Is 2d Over Niger Insurance*	*Inconclusive*	*Nem Insurance Is 2d Over Regency Assurance*
Niger Insurance	*Nem Insurance Is 2d Over Niger Insurance*		*Prestige Assurance Is 2d Over Niger Insurance*	*Inconclusive*
Prestige Assurance	*Inconclusive*	*Prestige Assurance Is 2d Over Niger Insurance*		*Inconclusive*
Regency Assurance	*Nem Insurance Is 2d Over Regency Assurance*	*Inconclusive*	*Inconclusive*	

Source: Author's computation using Vose Software (ModelRisk, 2020)

Table 4.1C: Dominance Analysis of Insurance Companies by Financial Performance cont'd

Dominance	***Royal Exchange***	***Sovereign Trust Insurance***	***Staco Insurance***	***Standard Alliance Insurance***
Royal Exchange		*Sovereign Trust Insurance Is 1d Over Royal Exchange*	*Staco Insurance Is 1d Over Royal Exchange*	*Royal Exchange Is 2d Over Standard Alliance Insurance*
Sovereign Trust Insurance	*Sovereign Trust Insurance Is 1d Over Royal Exchange*		*Sovereign Trust Insurance Is 2d Over Staco Insurance*	*Sovereign Trust Insurance Is 2d Over Standard Alliance Insurance*
Staco Insurance	*Staco Insurance Is 1d Over Royal Exchange*	*Sovereign Trust Insurance Is 2d Over Staco Insurance*		*Staco Insurance Is 2d Over Standard Alliance Insurance*
Standard Alliance Insurance	*Royal Exchange Is 2d Over Standard Alliance Insurance*	*Sovereign Trust Insurance Is 2d Over Standard Alliance Insurance*	*Staco Insurance Is 2d Over Standard Alliance Insurance*	
Dominance	***Sunu Assurances Nigeria***	***Universal Insurance Company***	***Veritas Kapital Assurance***	***Wapic Insurance***
Sunu Assurances Nigeria		*Universal Insurance Company is 1d over Sunu Assurances Nigeria*	*Veritas Kapital Assurance is 2d over Sunu Assurances Nigeria*	*Wapic Insurance is 1d over Sunu Assurances Nigeria*
Universal Insurance Company	*Universal Insurance Company is 1d over Sunu Assurances Nigeria*		*Inconclusive*	*Universal Insurance Company is 2d over Wapic Insurance*
Veritas Kapital Assurance	*Veritas Kapital Assurance is 2d over Sunu Assurances Nigeria*	*Inconclusive*		*Veritas Kapital Assurance is 2d over Wapic Insurance*
Wapic Insurance	*Wapic Insurance is 1d over Sunu Assurances Nigeria*	*Universal Insurance Company is 2d over Wapic Insurance*	*Veritas Kapital Assurance is 2d over Wapic Insurance*	

Source: Author's computation using Vose Software (ModelRisk, 2020)

4.2 Dominance Analysis of the Top Ten Performing Insurance Companies in Nigeria

In the preceeding section, the stochastic dorminance analysis of the twenty-four insurance companies were succintly considered. Thus, in this section, a critical analysis of the top ten financially performing insurance companies will be established. Essentially, two major criteria were adopted in the determination of the best financially thriving insurance firms captured in this study. The first criterion is the 11-year performance rate, which explains the extent to which these firm have achieved stability in profit making, while the second approach borders on the 11 year average performance of each of the listed insurance firms considered in this study.

Thus, the first 10 insurance companies with the highest performance rate, complemented by the 11-year average performance, were selected for further commparative assessments. Specifically, Figure 4A presents the comparative assesment outcomes of all the insurance firms. However, it was realised from the results that, Lasaco Assurance, Sovereign Trust Insurance, Consolidated Hallmark Insurance, Axamansard Insurance, Nem Insurance, Prestige Assurance, Staco Insurance, Aiico Insurance, Mutual Benefits Assurance and Regency Assurance emerged the top 10 performing insurance companies in Nigeria for the period 2005 to 2015. Figure 4A reports the performance assessment of the listed insurance companies in Nigeria below.

Figure 4A: Performance assessment of the listed insurance companies in Nigeria

LISTED INSURANCE FIRMS	11 years performance rate	11 Year Average
Lasaco Assurance	100.00%	428618
Sovereign Trust Insurance	100.00%	425969
Consolidated Hallmark Insurance	90.91%	1222763
Axamansard Insurance	90.91%	1131201
Nem Insurance	90.91%	476940
Prestige Assurance	90.91%	360203
Staco Insurance	90.91%	204224
Aiico Insurance	81.82%	690621
Mutual Benefits Assurance	81.82%	353461
Regency Assurance	81.82%	233662
Law Union And Rock Insurance	81.82%	89742
Universal Insurance Company	72.73%	295299
Niger Insurance	72.73%	258849
Veritas Kapital Assurance	72.73%	128364
Wapic Insurance	72.73%	108393
Cornerstone Insurance	72.73%	-78855
Linkage Assurance	63.64%	59283
Guinea Insurance	63.64%	-49417
Standard Alliance Insurance	63.64%	-1114333
International Energy Insurance	54.55%	-100938
Goldlink Insurance	45.45%	-183610
Sunu Assurances Nigeria	36.36%	-131088
Royal Exchange	27.27%	-40925
African Alliance Insurance	9.09%	-2222862

Source: Author's computation from the underlying data, using Microsoft Project (Excel, 2020)

Explicitly, Table 4.1D reports the dominance analysis of the top 10 performing insurance companies in Nigeria (Lasaco Assurance, Sovereign Trust Insurance, Consolidated Hallmark Insurance, Axamansard Insurance, Nem Insurance, Prestige Assurance, Staco Insurance, Aiico Insurance, Mutual Benefits Assurance and Regency Assurance). From the result, Lasaco Assurance is first-order stochastically dominant over Regency Assurance and second-order stochastically dominant over Sovereign Trust Insurance, Prestige Assurance and Staco Insurance.

Likewise, Sovereign Trust Insurance is second-order stochastically dominant over Staco Insurance and Regency Assurance, while Mutual Benefits Assurance is second-order stochastically dominant over Sovereign Trust Insurance. Similarly, Axamansard Insurance is first-order stochastically dominant over Nem Insurance and Regency Assurance, and second-order stochastically dominant over Prestige Assurance, Staco Insurance, Aiico Insurance and Mutual Benefits Insurance, while Nem Insurance is second-order stochastically dominant over Staco Insurance, Mutual and Regency Assurance. However, Aiico Insurance is second-order stochastically dominant over Staco Insurance and Mutual Benefits Insurance, while Regency Assurance is second-order stochastically dominant over Staco Insurance.

Nevertheless, the results returned inconclusive in the case of Lasaco Assurance and Consolidated Hallmark, Lasaco Assurance and Axamansard Insurance, Lasaco Assurance and Nem Insurance, Lasaco Assurance and Aiico Insurance, as well as Lasaco Assurance and Mutual Benefits Insurance. The case was similar for Consolidated Hallmark Insurance and Consolidated Hallmark, Consolidated Hallmark Insurance and Axamansard Insurance, Consolidated Hallmark Insurance and Nem Insurance, Consolidated Hallmark Insurance and Prestige Assurance, Consolidated Hallmark Insurance and Staco Insurance, Consolidated Hallmark Insurance and Aiico Insurance, Consolidated Hallmark Insurance and Mutual Benefits Insurance, as well as Consolidated Hallmark Insurance and Regency Assurance.

The results further returned inconclusive in the case of Sovereign Trust Insurance and Consolidated Hallmark, Sovereign Trust Insurance and Axamansard Insurance, Sovereign Trust Insurance and Nem Insurance, Sovereign Trust Insurance and Prestige Assurance, Sovereign Trust Insurance and Aiico Insurance, Nem Insurance and Prestige Assurance, Nem Insurance and Aiico Insurance, Prestige Assurance and Aiico Insurance, Prestige Assurance and Regency Assurance, Mutual Benefits Assurance and Staco Insurance, Regency Assurance and Aiico Insurance as well as Regency Assurance and Mutual Benefits Insurance.

Therefore, it can be concluded that, Axamansard Insurance stochastically dominates the most (6 times). This is closely followed by Lasaco Assurance (4times), Nem Insurance (3times) as well as Sovereign Trust Insurance, Prestige Assurance and Aiico Insurance (2times each). Nevertheless, one cannot differentiate and thus, remain indifferent between Lasaco Assurance and Consolidated Hallmark, Lasaco Assurance and Axamansard Insurance, Lasaco Assurance and Nem Insurance, Lasaco Assurance and Aiico Insurance, Lasaco Assurance and Mutual Benefits Insurance.

Same is true for Consolidated Hallmark Insurance and Consolidated Hallmark, Consolidated Hallmark Insurance and Axamansard Insurance, Consolidated Hallmark Insurance and Nem Insurance, Consolidated Hallmark Insurance and Prestige Assurance, Consolidated Hallmark Insurance and Staco Insurance, Consolidated Hallmark Insurance and Aiico Insurance, Consolidated Hallmark Insurance and Mutual Benefits Assurance, as well as Consolidated Hallmark Insurance and Regency Assurance. The results are reported in Table 4D.

Table 4.1D: Dominance Analysis of the Ten Best Performing Insurance Companies in Nigeria

Dominance	*Lasaco*	*Sovereign Trust*	*Consolidated Hallmark*	*Axamansard*	**Nem**	**Prestige**	**Staco**	**Aiico**	**Mutual Benefits**	**Regency**
Lasaco		*Lasaco is 2d over Sovereign Trust*	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Lasaco is 2d over Prestige*	*Lasaco is 2d over Staco*	*Inconclusive*	*Inconclusive*	*Lasaco is 1d over Regency*
Sovereign Trust	*Lasaco is 2d over Sovereign Trust*		*Inconclusive*	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Sovereign Trust is 2d over Staco*	*Inconclusive*	*Mutual Benefits is 2d over Sovereign Trust*	*Sovereign Trust is 2d over Regency*
C. Hallmark	*Inconclusive*	*Inconclusive*		*Inconclusive*	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Inconclusive*
Axamansard	*Inconclusive*	*Inconclusive*	*Inconclusive*		*Axamansard is 1d over Nem*	*Axamansard is 2d over Prestige*	*Axamansard is 2d over Staco*	*Axamansard is 2d over Aiico*	*Axamansard is 2d over Mutual Benefits*	*Axamansard is 1d over Regency*
Nem	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Axamansard is 1d over Nem*		*Inconclusive*	*Nem is 2d over Staco*	*Inconclusive*	*Nem is 2d over Mutual Benefits*	*Nem is 2d over Regency*
Prestige	*Lasaco is 2d over Prestige*	*Inconclusive*	*Inconclusive*	*Axamansard is 2d over Prestige*	*Inconclusive*		*Prestige is 2d over Staco*	*Inconclusive*	*Prestige is 2d over Mutual Benefits*	*Inconclusive*
Staco	*Lasaco is 2d over Staco*	*Sovereign Trust is 2d over Staco*	*Inconclusive*	*Axamansard is 2d over Staco*	*Nem is 2d over Staco*	*Prestige is 2d over Staco*		*Aiico is 2d over Staco*	*Inconclusive*	*Regency is 2d over Staco*
Aiico	*Inconclusive*	*Inconclusive*	*Inconclusive*	*Axamansard is 2d over Aiico*	*Inconclusive*	*Inconclusive*	*Aiico is 2d over Staco*		*Aiico is 2d over Mutual Benefits*	*Inconclusive*
Mutual Benefits	*Inconclusive*	*Mutual Benefits is 2d over Sovereign Trust*	*Inconclusive*	*Axamansard is 2d over Mutual Benefits*	*Nem is 2d over Mutual Benefits*	*Prestige is 2d over Mutual Benefits*	*Inconclusive*	*Aiico is 2d over Mutual Benefits*		*Inconclusive*
Regency	*Lasaco is 1d over Regency*	*Sovereign Trust is 2d over Regency*	*Inconclusive*	*Axamansard is 1d over Regency*	*Nem is 2d over Regency*	*Inconclusive*	*Regency is 2d over Staco*	*Inconclusive*	*Inconclusive*	

Source: Author's computation using Vose Software (ModelRisk, 2020)

5.0 Conclusion and Recommendations

This study undertakes a comparative performance analysis of listed insurance companies in Nigeria for the period 2005 – 2015, using stochastic dominance analysis. For all intents and purposes, the stochastic dominance analysis of the twenty-four (24) listed insurance firms in Nigeria was carried out based on their profit or loss for the period of study. Also, the top ten (10) insurance companies with the highest performance rate, complemented by the 11-year average performance, were selected for further comparative assessments. The comparative assessment results show that, Lasaco Assurance, Sovereign Trust Insurance, Consolidated Hallmark Insurance, Axamansard Insurance, NEM Insurance, Prestige Assurance, Staco Insurance, Aiico Insurance, Mutual Benefits Assurance and Regency Assurance emerged the top ten (10) performing insurance companies in Nigeria for the period of study. The findings of the stochastic dominance analysis revealed inter alia that, Axamansard Insurance stochastically dominates the Nigerian insurance market. This is closely followed by Lasaco Assurance, NEM Insurance as well as Sovereign Trust Insurance, Prestige Assurance and Aiico Insurance respectively. The empirical findings of this study suggest three likely circumstances that can drive the decision of any rational economic agent.

First, the individual or organisation intending to take an insurance cover may settle for a particular insurance firm sequel to the projected advantage, notwithstanding of the stringent conditionalities associated with the company under consideration. In this regard, such individual or organisation will tend to go for the firm(s) that first-order stochastically dominate(s) the most, given all conceivable pair comparison.

Second, the individual or organisation may be a risk-averse entity, who wishes to avoid any form of insurance policy uncertainty, regardless of the benefit of that insurance cover. In this regard, such individual or organisation will go for the firm or firms that second-order stochastically dominate (s) the most.

Finally, there are situations that neither makes the individual or organisation better-off, nor worse-off. In this regard, the individual or organisation will remain indifferent. This occurs when the outcomes are returned inconclusive. In this respect, the individual or organisation can choose either of the pair with no relative advantage.

References

Bernoulli, D. (1738). Specimen theoriae novae de mensura sortis.Commen-tarii academaiae scientiarium imperiales petropolitanae, *5*(I), 75–192. (Translated by L. Sommer in Econometrica, 1954, 22, 23–36).

Bibi S., Duclos J.Y., & Audrey V. C. (2012). Assessing Absolute and Relative Pro-Poor Growth, with an Application to Selected African Countries. Economics: *The Open-Access, Open-Assessment E-Journal,* 6(2), 1−43.

Blavaskyy, P. R. (2010). Modifying the mean-variance approach to avoid violations of stochastic dominance. *Management Science 56*(11), 2050−2057.

Blavaskyy, P. R. (2011). A model of probabilistic choice satisfying first-order stochastic dominance. *Management Science,* 57(3), 542−548.

Chiu, W. H. (2005). Degree of downside risk aversion and self-protection. Insurance: *Mathematics and Economics, 36*(1), 93−101.

Deutsch J. & Silber J. (2011). On Various Ways of Measuring Pro-Poor Growth. *Economics: The Open-Access, Open-Assessment E-Journal 5*(2), 1−57.

Francisco, P. (2007). Portfolio choice beyond the traditional approach. Discussion paper No 587.

Hadar, J., W. Russell, W.R., (1969). Rules for ordering uncertain prospects. *American Economic Review, 59*(1), 25-34

Hammond, J. S. (1974). Simplifying the choice between uncertain prospects where preference is nonlinear. *Management Science*, 20(7), 1047-1072.

Hanoch, G. & Levy, H. (1969). The efficiency analysis and choices involving risk. *The Review of Economic Studies*, *36*(3), 335–346.

Heyer, D. D. (2001). Stochastic dominance: a tool for evaluating reinsurance alternatives, in: Casualty Actuarial Society Forum, 95 - 118.

Kopa, M. & Chovanec, P. (2008). A second - order stochastic dominance portfolio efficiency measure. *Kybernetika, 44(2), 243-258.*

Kopa, M. & Chovanec, P. (2008). A second - order stochastic dominance portfolio efficiency measure. *Kybernetika, 44(2), 243-258.*

Lee, Y.S. (1988). The mathematics of excess of loss coveraaes and retrospective ratino - A graphical approach. *PCAS LXXV.*

Lehmann, E. L. (1955). Ordered families of distributions. *Annals of Mathimatical. Statist.*, 26, 399–419.

Leili, J. (2013). Portfolio selection by second order stochastic dominance based on the risk aversion degree of investors. University of Toronto, Toronto.

Levy, H. (1992). Stochastic dominance and expected utility: Survey and analysis. *Management Science*, *38*(4), 555−593.

Levy, H. (2006). *Stochastic dominance: Investment decision making under uncertainty*. 2nd edition. Kluwer Academic, Boston.

Li, C.K., & Wong, W. K. (1999). A note on stochastic dominance for risk averters and risk takers. *RAIRO Recherche Opérationnelle*, *33*(5), 509-524.

Li, J. (2009). Comparative higher-degree Ross risk aversion. Insurance: *Mathematics and Economics*, *45*(3), 333−336.

Lizyayev, A.M. (2010). *Stochastic Dominance in Portfolio Analysis and Asset Pricing* (No. 487). *Tinbergen Instituut Research Series*. Erasmus MC: University Medical Center Rotterdam. Retrieved from http://hdl.handle.net/1765/21330

Loomes, G., Inmaculada, R.P., & Pinto-Prades, J. L. (2014). Comment on "A model of probabilistic choice satisfying first-order stochastic dominance". *Management Science, 60*(5), 1346 − 1350.

Mann, H. B., & Whitney, D. R., (1947). On a test of whether one of two random variables is stochastically larger than the other. *Annals of Mathematical Statistics, 18*(1), 50 – 60.

Markowitz, H. (1952). Portfolio selection. *Journal of finance,* 7(1), 77 - 91.

Meyer, J. (1989). *Stochastic dominance and transformations of random variables*. In Thomas B. Fomby and Tae Kun Seo (Eds.), Studies in the economics of uncertainty. In Honor of Josef Hadar, Springer Verlag, New York.

Osifo, O. (2018). Stochastic dominance and portfolio selection in Nigeria. (Unpublished Ph.D seminar paper). University of Benin: Department of Banking and Finance.

Post, T., & Kopa, M. (2015). Portfolio choice based on third-degree stochastic dominance, with an application to industry momentum. *KOC University TUSIAD Economic Working Paper*, 1527.

Quirk, J. P., & Saposnik, R. (1962). Admissibility and measurable utility functions. *The Review of Economic Studies, 29*(2), 140-146.

Tsetlin, I., Winkler, R. L., Huang, R.J., & Tzeng L. Y. (2015). Generalized almost stochastic dominance. *Operations Research*, *63*(2), 363−377.

Tzeng, L.Y., Huang, R.J., & Shih, P.T. (2013). Revisiting almost second-degree stochastic dominance. *Management Science*, *59*(5), 1250−1254.

Valentini E. (2015). Indirect taxation, public pricing and price cap regulation: A synthesis. *Economics: The Open-Access, Open-Assessment E-Journal*, 9, 1−39.

Wolfstetter, E. G. (1996). Stochastic dominance: Theory and application. Retrieved from https://www.researchgate.net/publication/2290304

Yalonetzky G. (2012). Measuring group disadvantage with inter-distributional inequality indices: A critical review and some amendments to existing indices. *Economics: The Open-Access, Open-Assessment E-Journal*, *6*(3), 1 - 32.

Wong, W. (2007). Stochastic dominance and mean–variance measures of profit and loss for business planning and investment. *European Journal of Operational Research, 182* (2), 829-843. https://doi.org/10.1016/j.ejor.2006.09.032.

Jianwei, G. & Feng, Z (2017). A new approach of stochastic dominance for ranking transformations on the discrete random variable. Economics: *The Open-Access, Open-Assessment E-Journal, 11* (2017-14), 1–23. http://dx.doi.org/10.5018/economics-ejournal.ja.2017-14